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VELOCITY OF MOTION OF BODIES WITH TOSSING ON A VIBRATING SURFACE

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A new method is suggested for studying and designing vibrational-transportation mechanisms. The method allows one to find the optimum geometric and physical characteristics of the component parts of machines at a preliminary stage of their development using the conditions of specified velocity parameters of the vibrational motion of the bodies with tossing.

One of the important stages in the study of the mechanisms that transport bodies with tossing on an inclined vibrating surface is the determination of the velocity of directed motion of material bodies. These mechanisms must provide high efficiency and reliability, operate in elevated dynamic modes, and destroy, separate, or transport material layers, objects, bodies, or particles. For this purpose we conducted theoretical studies that differ from widely known ones and we derived basic analytical relations, some of which are given in [1]. The mechanism for transportation of bodies with tossing on a vibrating surface is shown in Fig. 1. Its kinematic characteristics are:

$$v_{Px} = -r\omega \frac{\sin(\varphi + \theta_1)}{\cos(\varphi_1 - \theta_1)} \cos\varphi_1 , \tag{1}$$

$$v_{Py} = r\omega \frac{\sin(\varphi + \theta_1)}{\cos(\varphi_1 - \theta_1)} \sin\varphi_1 , \tag{2}$$

$$a_{Px} = -\omega^2 r \cos\varphi_1 \frac{\cos(\varphi + \theta_1)}{\cos(\varphi_1 - \theta_1)} - \frac{\omega^2 r^2}{l_3} \frac{\cos^2(\varphi + \varphi_1)}{\cos^3(\varphi_1 - \theta_1)} \cos\varphi_1 + \frac{\omega^2 r^2}{l_4} \frac{\sin^2(\varphi + \theta_1)}{\cos^3(\varphi_1 - \theta_1)} \sin\theta_1 , \tag{3}$$

$$a_{Py} = \omega^2 r \sin\varphi_1 \frac{\cos(\varphi + \theta_1)}{\cos(\varphi_1 - \theta_1)} + \frac{\omega^2 r^2}{l_3} \frac{\cos^2(\varphi + \varphi_1)}{\cos^3(\varphi_1 - \theta_1)} \sin\varphi_1 + \frac{\omega^2 r^2}{l_4} \frac{\sin^2(\varphi + \theta_1)}{\cos^3(\varphi_1 - \theta_1)} \cos\theta_1 , \tag{4}$$

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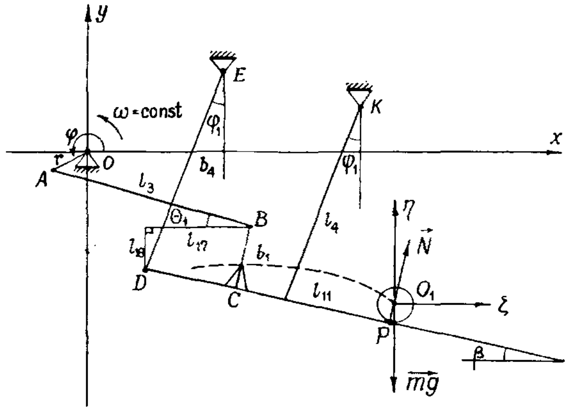

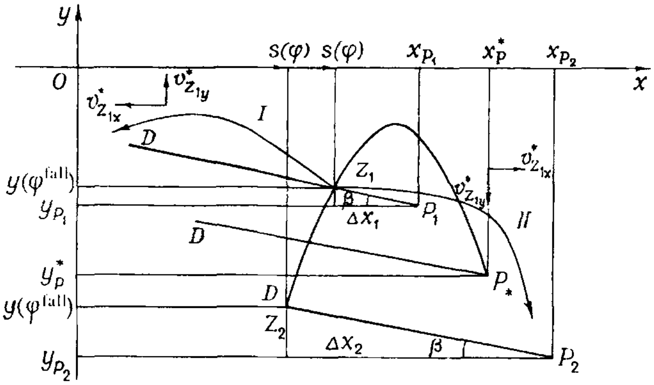

Fig. 1. Schematic of a reciprocating tray *DP* with a body of mass *m*.

Fig. 2. Trajectory of flight of a material body.

$$\varphi_1(\varphi) = \arctan\frac{(r\sin\varphi - d)}{(r\cos\varphi - a)} +$$

$$+ \arcsin\left(\frac{l_4^2 + r^2 + a^2 + d^2 - 2r(a\cos\varphi + d\sin\varphi)}{2l_4\sqrt{(r\cos\varphi - a)^2 + (r\sin\varphi - d)^2}}\right), \tag{5}$$

The point P is displaced in coordinate form according to the expressions

$$x_P = x_E - l_4\sin\varphi_1 + l_{16}\cos\beta, \tag{6}$$

$$y_P = y_E - l_4\cos\varphi_1 - l_{16}\sin\beta. \tag{7}$$

The instant t_Δ of separation of a material body from the surface of the tray at the point P, the kinematic characteristics, and the corresponding angle of rotation of the crankshaft OA were specified in [1]. Under these conditions a body rises above the surface DP and flies at an initial velocity from a position denoted by an asterisk. In a new problem of ballistics theory, these values of the velocity and the coordinates are constant. The trajectory of the body between the rise and the fall in coordinate form is as follows:

$$y(\varphi) = y_P^* + v_{Py}^*\Delta t - \frac{g\Delta t^2}{2}, \tag{8}$$

$$x(\varphi) = x_P^* + v_{Px}^*\Delta t, \tag{9}$$

where $\Delta t = t - t_\Delta$, $\Delta\varphi = \varphi - \varphi^* = \omega\Delta t$; x_P^*, y_P^*, t_Δ, and φ^* are constants determined in [1].

The velocity of transportation of material bodies along the inclined plane depends substantially on the flying range $l = \Delta x$. Two variants of the fall of the body onto the plane DP during one period of vibrations of the crank OA are possible (Fig. 1): to the point Z_1 or Z_2 (Fig. 2). In the first case, $x_{P_1} - x_P^* < 0$, and in the second case, $x_{P_2} - x_P^* > 0$. The second variant is preferable since in this case the range of flying relative to the tray DP is larger, thus leading to an increase in the velocity of motion of the bodies on the vibrating surface in one revolution of the crank OA. As a result, we express the velocity of motion as

$$V = Sv, \tag{10}$$

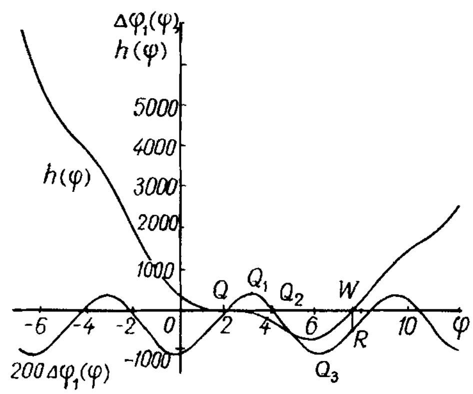

Fig. 3. Graphs of the functional dependence of $h(\varphi)$ and $200\Delta\varphi_1(\varphi)$ on φ. $h(\varphi)$, $\Delta\varphi_1(\varphi)$, deg; φ, rad

where

$$S = \Delta x \sqrt{1 + \tan^2 \beta} \tag{11}$$

is the flying range of the body in projection onto the plane DP, $\nu = \omega/2\pi$ is the cyclic frequency of the crank OA.

We consider both variants of determination of Δx (Fig. 2):

$$\Delta x_1 = x_{P_1} - s(\varphi), \quad \Delta x_2 = x_{P_2} - s(\varphi).$$

In these relations, $s(\varphi) = x(\varphi)$. Generalizing these formulas, we can find that

$$\Delta x = x_P - \overset{*}{x}_P - \overset{*}{v}_{Px} \Delta t. \tag{12}$$

With account for (6), we convert (12) to the form

$$\Delta x = l_4 (\sin \overset{*}{\varphi}_1 - \sin \varphi_1) - \overset{*}{v}_{Px} \Delta t. \tag{13}$$

For small variations of the angles $\Delta\varphi_1$, we write the approximate formula

$$\Delta x = - l_4 \cos \overset{*}{\varphi}_1 \Delta\varphi_1 - \overset{*}{v}_{Px} \Delta t, \tag{14}$$

where $\Delta\varphi_1 = \varphi_1^{\text{fall}} - \varphi_1^*$; $\Delta\varphi = \varphi^{\text{fall}} - \varphi^*$. The values of the angles φ_1^{fall} and φ^{fall} at the instant of the fall of the body onto the plane DP are determined from the condition of equality of the coordinate y of the flight height (8) and the straight line DP. It should be taken into account that the surface of the tray DP (Fig. 1) executes a reciprocating motion; its inclination during the entire motion remains constant. Therefore,

$$y(\varphi) = y_P(\varphi) + \tan\beta \, |\Delta x(\varphi)|. \tag{15}$$

In formula (15) we substitute the value (8) for $y(\varphi)$ and after certain manipulations we obtain a non-linear transcendental equation in the variables $\Delta\varphi_1$ and $\Delta\varphi$ for determination of φ^{fall}:

$$h(\varphi) = \Delta\varphi^2 - b\Delta\varphi - c(\cos\varphi_1 - \cos\overset{*}{\varphi}_1 + \tan\beta(\sin\varphi_1 - \sin\overset{*}{\varphi}_1)) = 0, \tag{16}$$

$$b = \frac{2\omega}{g}(\tan\beta \, \overset{*}{v}_{Px} + \overset{*}{v}_{Py}), \quad c = \frac{2\omega^2 l_4}{g}.$$

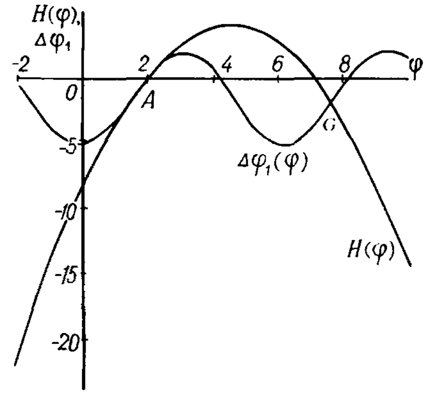

Fig. 4. Graphs of the dependence of $H(\varphi)$ and $\Delta\varphi_1$ on φ. $H(\varphi)$, deg.

The solution of Eq. (16) can easily be found numerically. We give an example of its realization for mechanisms with the physical characteristics used (Fig. 3).

In Fig. 3, the instant of separation is denoted by the point Q and the instant of fall by W. To derive simple analytical relations that are convenient in practice in engineering calculations, it is more expedient to replace formula (16) by an approximate formula, neglecting small quantities of higher order compared to $\Delta\varphi_1$:

$$\Delta\varphi^2 - b\Delta\varphi - c_1\Delta\varphi_1 = 0\,, \quad c_1 = c\,(\tan\beta\cos\varphi_1^* - \sin\varphi_1^*)\,, \tag{17}$$

from which

$$\Delta\varphi_1\,(\varphi) = \frac{1}{c_1}\,\Delta\varphi\,(\Delta\varphi - b)\,. \tag{18}$$

The right-hand side of expression (18) and the known function $\varphi_1(\varphi)$ of (5) are shown in Fig. 4:

$$H\,(\varphi) = \frac{1}{c_1}\,(\varphi - \varphi^*)\,(\varphi - \varphi^* - b)\,. \tag{19}$$

The points of intersection of the graphs (Fig. 4) are the roots of the nonlinear equation (18) and in Fig. 3 correspond to

$$\varphi_A \rightarrow \varphi_Q\,, \quad \varphi_G \rightarrow \varphi_W\,.$$

Here $\varphi_A = \varphi(A)$ and $\varphi_G = \varphi(G)$ are the values of the angles φ at the points A and G depicted in Fig. 4; $\varphi_Q = \varphi(Q)$ and $\varphi_W = \varphi(W)$ are the values of the angles φ at the points Q and W (Fig. 3). In our example, as φ_1 changes within the limits from 28.6 to 35.8°, $\varphi_A = 1.9720$ rad, $\varphi_Q = 2.0180$ rad, $\varphi_G = 7.6552$ rad, and $\varphi_W = 7.6415$ rad. The relative error is 0.18–2.3%.

We replace $\Delta\varphi_1$ in (14) by the approximate analytical expression (18), and then we have

$$\Delta x = -\,l_4\,\frac{1}{c_1}\cos\varphi_1^*\,\Delta\varphi\,(\Delta\varphi - b) + \frac{1}{\omega}\,|v_{Px}^*|\,\Delta\varphi\,. \tag{20}$$

This relation involves the negative constant value of the projection of the velocity v_{Px}^* at the instant of separation [1], and therefore, without loss of the generality of the considerations we can take a minus sign outside the modulus:

$$v_{Px}^* = -\,|v_{Px}^*|\,.$$

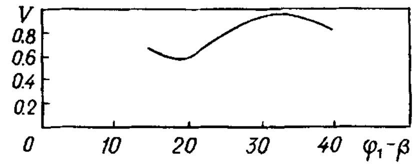

Fig. 5. Graph of the dependence of the velocity of vibrational motion V of material bodies on the difference of angles $\varphi_1 - \beta$ at $\beta = 13°$. V, m/sec; $\varphi_1 - \beta$, deg.

An analysis of formulas (10) and (13), (14), (20) for the velocity of motion of material bodies along the inclined vibrating surface DP leads to the conclusion that upon the fall of the bodies to the point Z_1 (Fig. 2) their velocity is smaller, since the first term in (13), (14), and (20) is less than zero. Upon the fall of the bodies to the point Z_2 the velocity is larger, since at this instant the tray DP executes backward motion relative to the motion of the material body; this is expressed in the summation of two positive terms in formulas (13), (14), and (20). In practice, use of a mechanism in which the take-off and the fall occur during one revolution of the crank OA or one period of the vibration $\Delta\varphi = 2\pi$ is most rational [2-6]. Therefore, in the flying range of material bodies the best effect can be achieved by selecting geometric parameters and physical characteristics that provide a maximum of the function $\Delta x(\varphi)$ in formula (13). Two interrelated features are typical of Δx. The first must provide a $\Delta\varphi_1$ of (14) and (20) that is maximum in magnitude and negative in value (the point R in Fig. 3). Physically this means that after separation of the body from the surface DP at the point P it must fall at an instant when the plane DP executes backward motion or almost one full vibration up to its extreme position corresponding to the right dead point Q_3 (Fig. 3).

The second feature is based on determination of the maximum value of $\Delta\varphi$ among the entire set of possible motions of a mechanism with specified geometric and kinematic properties. Here, $\Delta\varphi$ is limited only by the period of the vibrations 2π. In this case the angular velocity ω and the angle of slope β of the plane DP to the horizontal can be varied. As the angular velocity ω increases $v^*_{P_x}$, $v^*_{P_y}$, and $\Delta\varphi$ increase, correspondingly. A more complex dependence exists between the velocity of transportation and the angle of slope β. For example, for our data it is represented by a graph (Fig. 5) that is in good agreement with results of experiments conducted using high-speed filming [6].

A number of obvious conclusions can be drawn proceeding from explicit formulas (1), (2), (8), (9), and (20). An increase in the angle of slope φ_1 of the suspensions l_4 to the vertical leads to an increase in the flying height and range of material bodies. This follows from an analysis of formulas (1) and (2) for rather small angles θ_1. Then the components of the velocity of the material body at the instant of separation are

$$\overset{*}{v}_{Px} = -\,r\omega \sin \varphi_* \,, \quad \overset{*}{v}_{Py} = r\omega \tan \overset{*}{\varphi_1} \sin \varphi_* \,.$$

According to well-known laws of ballistics, the body reaches a maximum flying range at $v^*_{P_y}/|v^*_{P_x}| = 1$ or, which is the same, $\tan \varphi_1^* = 1$, and consequently, $\varphi_1 \to 45°$. However, at the same time the condition $\Delta\varphi_1^* < 0$ must be met. Therefore, an excessive increase in φ_1 leads to negative results. Mechanisms in which φ_1 varies from 25 to 38° possess the best characteristics.

We pay attention to a number of important facts. Achievement of an increase in the velocity of transportation by changing the geometric and physical parameters may cause the bodies to fall onto a plane DP that executes a vibration according to law (6) and (7) that corresponds to the portion QQ_2 of the curve $\Delta\varphi_1$ (Fig. 3) or the upper part of the hodograph of the inertial force above W (Fig. 2 in [1]). At the point Q the body rises above the surface of a tray that continues to move upward (Fig. 1) and reaches its extreme left position – the dead point Q_1. Here its velocity is zero. Then, from the point Q_1 the surface DP begins to execute backward motion; here, its velocity increases but has the opposite direction. Then, if the body falls onto DP at an instant of motion that graphically corresponds to the section QQ_1, then, having received a rather large impulse, it recoils from the surface, according to the direction of the velocity, upward to the left (trajectory I, Fig. 2). If the body arrives at a DP that at this instant executes backward motion downward from Q_1 to Q_2 (Fig.

3), then it receives an impulse which repells it downward (trajectory *II*, Fig. 2), and the material objects are unloaded.

Mathematical modeling of vibrational-transportation devices that provide directed motion of loads with tossing is a very complex analytical problem that requires consideration of many parameters and operational nuances. In spite of available experience [2-7] in the design of these mechanisms and a wide range of experimental data, each specific mechanism should be studied individually. The technique suggested leads to simple analytical dependences between the geometric and kinematic parameters and makes it possible to establish their influence on the physical process as a whole.

NOTATION

r, φ, and ω, radius, angle of rotation, and angular velocity of the crank; t, time coordinate; l_i, $i = \overline{1, 18}$, h_1, lengths of the elements of the crank mechanism (Fig. 1); $l_{16} = l_5 + l_{11}$; $\theta_1(t)$ and $\varphi_1(t)$, angles of rotation of the elements l_3 and l_4; x_E, y_E, x_P, and y_P, coordinates of the points E and P; m, mass of the material object; g, free-fall acceleration; $\eta\zeta$, coordinate system related to the surface of the tray; β, angle of slope of the surface of the tray DP to the horizontal; $v_{P_x}(t)$, $v_{P_y}(t)$ and $a_{P_x}(t)$, $a_{P_y}(t)$, velocities and accelerations of the material object on the tray at the point P; V, velocity of the vibrational motion; $h(\varphi)$ and $H(\varphi)$, intermediate functions of the parameter φ.

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